

# Common Openhole Petrophysics Interpretation Equations

## Comments

The equations on the following pages convert the measurements commonly made in open (uncased) boreholes into the formation properties of scientific and economic interest to us; those being porosity, fluid saturation, and lithology. There are also equations which take us from some of the measurements to calculation parameters that are needed in some of the equations, such as formation water resistivity,  $R_w$ .

The listing of the equations follows a generic workflow. In this workflow, the order of the steps may sometimes change, and all the workflow steps may not be used. The steps in the workflow and the measurements used are as follows:

Workflow	Measurement group:
<ul style="list-style-type: none"><li>• Is the data valid?</li><li>• Where is the reservoir?</li></ul>	Measurements <i>Correlation/Reservoir:</i> Gamma ray, SP, Caliper, Tension
<ul style="list-style-type: none"><li>• Is there sufficient porosity?</li><li>• Is the lithology different than expected, or varying in some way?</li></ul>	<i>Porosity and Lithology:</i> Acoustic slowness, bulk density, neutron porosity. Combinations of acoustic slowness, bulk density, photoelectric effect, neutron porosity.
<ul style="list-style-type: none"><li>• What are the fluids in place?</li><li>• What fluids will be produced?</li></ul>	<i>Resistivity/Fluid saturation:</i> Induction and laterologs, both close to and away from the borehole, with porosity.

The **Measurement Groups**, above, are how the measurements are grouped in two courses that have been taught (with revisions) for decades: *Basic Well Log Analysis* and *Basic Openhole Log Interpretation*. The groupings are based on the targets or goals of the measurements, and not the physics of the measurements, as the measurement targets are more in line with the general workflow.

The references are listed on the last page of this document.

*This document is intended to be updated periodically as necessary to include new and corrected information.*

*Questions and comments about this document are welcomed and encouraged. Please contact Dan Krygowski at The Discovery Group; DanKrygowski@Discovery-Group.com.*

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## Determination of formation temperature

$$FmTemp = \left( \frac{BHT - AMST}{TD} * FD \right) + AMST$$

Where: BHT = Bottom Hole Temperature  
 TD = Total Depth; FD = Formation Depth  
 AMST = Annual Mean Surface Temperature

## Temperature conversion of resistivity

$$R_{FmTemp} = \frac{R_{MeasTemp} * (MeasTemp + C)}{FmTemp + C}$$

Where: MeasTemp = Measured Temperature  
 FmTemp = Formation Temperature  
 For °F, C = 6.77; for °C, C = 21.3

*Arps, 1953*

## Formation water resistivity, $R_w$

### From the SP

*First order: Wyllie, 1949*

$$R_w = 10^{(K \cdot \log(R_{mf}) + SP) / K}$$

Where  $K = 61 + 0.133 * FmTemp$

*Rweq method (preferred): Gondouin et al, 1957*

$$R_{weq} = R_{mf} * 10^{\left( \frac{SSP}{61 + 0.133 * Fm.Temp} \right)}$$

$$R_w = \frac{R_{weq} + 0.131 * 10^{[1/\log(Fm.Temp/19.9)] - 2}}{-0.5 * R_{weq} + 10^{[0.0426/\log(Fm.Temp/50.8)]}}$$

## Shale/Clay Volume

### Spontaneous Potential

*(no reference found)*

$$V_{sh} = \frac{SP_{clean} - SP_{log}}{SP_{clean} - SP_{shale}}$$

### Neutron Porosity and Density Porosity

*(no reference found)*

$$V_{sh} = \frac{\Phi_{iN} - \Phi_{iD}}{\Phi_{iN_{shale}} - \Phi_{iD_{shale}}}$$

### Gamma Ray

*Linear: (no reference found)*

$$I_{GR} = V_{sh} = \frac{GR_{log} - GR_{clean}}{GR_{shale} - GR_{clean}}$$

*Steiber (1970):*

$$V_{sh} = \frac{I_{GR}}{(3.0 - 2.0 \cdot I_{GR})}$$

*Clavier (Clavier et al, 1971):*

$$V_{sh} = 1.7 - \left[ 3.38 - (I_{GR} + 0.7)^2 \right]^{\frac{1}{2}}$$

*Larionov (Tertiary) (1969):*

$$V_{sh} = 0.083 \cdot (2^{(3.7 \cdot I_{GR})} - 1.0)$$

*Larionov (older rocks) (1969):*

$$V_{sh} = 0.33 \cdot (2^{(2.1 \cdot I_{GR})} - 1.0)$$

Because logging measurements measure a volume in space, rather than a point in space, the SP and gamma ray usually measure "shale" rather than a specific clay. Given enough core information about the clays that are present, one can make that transition from working in shale-space to working in clay-space. Working in either "space" is adequate if one is consistent in the approach used.

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## Porosity

**Effective porosity (from any source) (no reference found)**

$$\Phi_E = \Phi_T - V_{sh} * \Phi_{Xsh}$$

where  $\Phi_T$  = Total porosity

$\Phi_{Xsh}$  = porosity in a nearby shale

**Bulk Density (no reference found)**

$$\Phi_D = \frac{RHO_{ma} - RHO_B}{RHO_{ma} - RHO_{fl}}$$

**Neutron (no reference found)**

Neutron porosity is referenced to a specific lithology. Conversion of porosity is dependent on the company that ran the log, the generation of the neutron tool, and often the specific curve name.

**Neutron and Density (no reference found)**

With both porosity curves referenced to limestone porosity units:

Liquid-filled holes:

$$\Phi_{IND} = \frac{\Phi_{N_{LS}} + \Phi_{D_{LS}}}{2}$$

Gas-filled holes:

$$\Phi_{IND} = \left( \frac{\Phi_{N_{LS}}^2 + \Phi_{D_{LS}}^2}{2} \right)^{\frac{1}{2}} \cong \left( \frac{1}{3} * \Phi_{N_{LS}} + \frac{2}{3} * \Phi_{D_{LS}} \right)$$

See also the Characteristic Logging Tool Responses table for additional matrix and fluid values.

## Sonic

Wyllie Time-Average:

Wyllie et al, 1958; Tixier et al, 1959

$$\Phi_S = \frac{DT - DT_{ma}}{DT_{fl} - DT_{ma}} * \frac{1}{B_{cp}} \quad B_{cp} = \frac{DT_{shale}}{100} \geq 1.0$$

Empirical/Field Observation:

Raymer et al, 1980; Baker Hughes, 2003

$$\Phi_S = -\alpha - \left[ \alpha^2 + \frac{DT_{ma}}{DT} + 1 \right]^{\frac{1}{2}} \quad \alpha = \frac{DT_{ma}}{2 * DT_{fl}} - 1$$

Empirical/Field Observation (simplified form):

$$\Phi_S = \frac{2}{3} * \frac{DT - DT_{ma}}{DT} \quad \text{AERA, 2007}$$

## Lithology from porosity logs

M-N plots Burke et al, 1969

$$M = \left( \frac{DT_{fl} - DT}{RHO_B - RHO_{fl}} \right) * C$$

$$N = \left( \frac{1 - \Phi_{Nls}}{RHO_B - RHO_{fl}} \right) * D$$

MID (Matrix IDentification) plots

$$DT_{maa} = \frac{DT - (\Phi_{INS} * DT_{fl})}{1 - \Phi_{INS}}$$

$$RHO_{maa} = \frac{RHO_B - (\Phi_{IND} * RHO_{fl})}{1 - \Phi_{IND}}$$

Clavier & Rust, 1976

$$U_{maa} = \frac{(P_e * RHO_B) - (\Phi_{IND} * U_{fl})}{1 - \Phi_{IND}}$$

Gardner & Dumanoir, 1980

Fluid values and constants		Bulk density		Sonic slowness		Neutron porosity	C	D
		g/cm <sup>3</sup>	Kg/m <sup>3</sup>	μsec/ft	μsec/m			
US Oilfield	fresh	1.0		189		1	0.01	1
	salt	1.1		189				
Metric	fresh	1.0			620	1	0.003048	1
	salt	1.1			620			
Canadian	fresh		1000		620	1	3.048	1000
	salt		1100		620			

Where...	Fresh water	Salt water
RHO <sub>fl</sub> (g/cm <sup>3</sup> )	1.0	1.2
DT <sub>fl</sub> (μsec/ft)	189	189
U <sub>fl</sub> (b/cm <sup>3</sup> )	0.398	1.36

See also the Characteristic Logging Tool Responses table for additional matrix and fluid values.

# Common Openhole Petrophysics Interpretation Equations

## Water Saturation

Apparent water resistivity,  $Rwa$  (no reference found)

$$Rwa = \frac{Rt * Phi^m}{a} \quad Sw = \left( \frac{Rwa_{\text{minimum}}}{Rwa_{\text{zone}}} \right)^{\frac{1}{n}}$$

Archie (1942)

$$Sw = \left[ \frac{a * Rw}{Rt * Phi^m} \right]^{\frac{1}{n}}$$

Bulk Volume Water, BVW

Buckles, 1965; Morris & Biggs, 1967

$$BVW = Phi * Sw$$

$$BVW_{irr} = Phi * Sw_{irr}$$

## Permeability from logs

Timur (1968)

$$K = 62500 * \frac{Phi^6}{Sw_{irr}^2}$$

where Phi and Sw are in v/v decimal.

Tixier (no reference found)

$$K = \left( \frac{250 * Phi^3}{Sw_{irr}} \right)^2$$

Coates-Timur (NMR)

(no reference found)

$$MPERM = \left[ \left( \frac{MPHI}{C} \right)^2 \left( \frac{FFI}{BVI} \right) \right]^2$$

C = constant

SDR (Schlumberger Doll Research) (NMR)

(no reference found)

$$PERM = a T_{2gm}^2 \cdot \phi^4$$

a = constant;  $T_{2gm}$  = T<sub>2</sub> geometric mean

## Mechanical Properties

Young's Modulus

$$E = \left( \frac{RHOB}{DTS^2} \right) \cdot \left( \frac{3DTS^2 - 4DTC^2}{DTS^2 - DTC^2} \right) \cdot 1.34 \times 10^{10}$$

Shear Modulus

$$\mu = \left( \frac{RHOB}{DT_S^2} \right) \cdot 1.34 \times 10^{10}$$

RHOB in g/cm<sup>3</sup>

DTS, DTC in μsec/ft

Bulk Modulus

$$K = RHOB \cdot \left( \frac{3DTS^2 - 4DTC^2}{3DTS^2 - DTC^2} \right) \cdot 1.34 \times 10^{10}$$

Poisson's Ratio

$$\sigma = \frac{1}{2} \cdot \left( \frac{DTS^2 - 2DTC^2}{DTS^2 - DTC^2} \right)$$

Kowalski, 1975

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## Equation References

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