The Interrelation Between Gas and Oil Relative Permeabilities

By A. T. Corey*

Introduction

The relative permeability of a reservoir rock to each of the fluids flowing through it is important in the prediction of reservoir behavior. If a direct relationship between gas and oil relative permeabilities could be demonstrated, considerable time would be saved in the analysis of large numbers of reservoir samples. Such a relationship would also contribute toward a better understanding of the fundamental aspects of fluid flow in porous material.

From the results of numerous measurements in this laboratory by the capillary pressure technique, a relationship was observed between gas and oil relative permeabilities. This paper presents a method for calculating oil relative permeabilities from measured gas relative permeabilities using the observed relationship. It also presents a rationalization of the relationship based on the Kozeny-Carman equation and the properties of the capillary pressure-desaturation function.

Historical Background

Experimental measurements to determine the conductivity of porous rock to fluids and the factors affecting it have long been recorded in the literature. One of the early attempts to interrelate these quantities in a rational manner resulted in the Kozeny-Carman equation. Later attempts have resulted in essentially the same relationship.

According to the Kozeny-Carman equation, permeability of a porous material can be expressed as a function of the product of two parameters one of which evaluates the effective path length of the flowing fluid and the other which evaluates the mean hydraulic radius of channels through which the fluid flows.

Rose and Bruce[1] suggested the use of the Kozeny-Carman relationship to predict relative permeability. The name tortuosity was given to the parameter evaluating the effective path length. In their original paper, however, it was assumed that saturation had little effect on tortuosity. Thornton[2] and later Wyllie and Rose[3] noted that tortuosity was saturation dependent and suggested its determination by electrical measurements.

Using a bundle of capillary tubes of varying sizes as an analogy, Purcell[4] developed a formula for the permeability of a porous system in terms of its porosity and its capillary pressure-desaturation curve. Gates and Lietz[5] applied this formula to partial saturations and introduced a tortuosity coefficient, e, determined empirically. Wyllie and Spangler[6] and later Burdine[7] derived an analogous formula by using the mean hydraulic radius concept of Kozeny-Carman for each pore size in a rock having a wide variety of pores. For a detailed discussion of this theory and its historical development, the reader is referred to the paper by Wyllie and Spangler.[6]

From measured relative permeabilities, Burdine[7] observed that the tortuosity parameters could be approximated by simple expressions in terms of fluid saturations. For all relative permeability, Burdine's equation can be expressed as:

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^2 \frac{\int_o^{S_o} dS_o/P_c^2}{\int_o^1 dS_o/P_c^2} \qquad (1)$$

and for gas relative permeability

$$K_{rg} = \left(1 - \frac{S_o - S_{or}}{S_m - S_{or}}\right)^2 \frac{\int_{S_o}^1 dS_o/P_c^2}{\int_o^1 dS_o/P_c^2} \qquad (2)$$

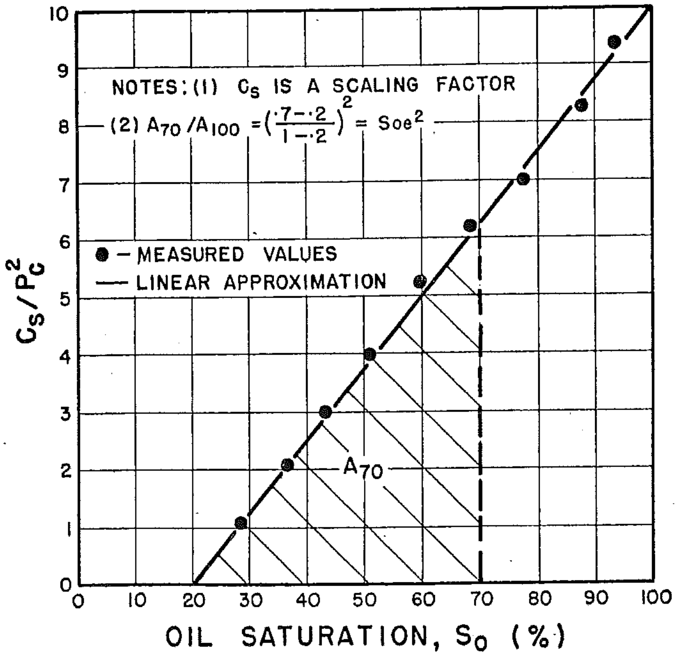

Figure 1—Plot of typical ($1/P_c^2$ vs S_o) function.

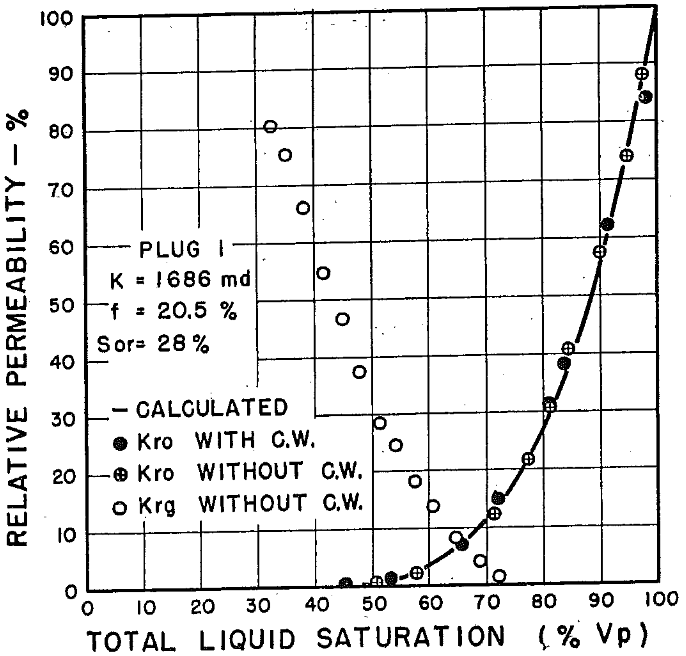

Figure 2—Relative permeabilities on consolidated sand showing agreement with calculated values.

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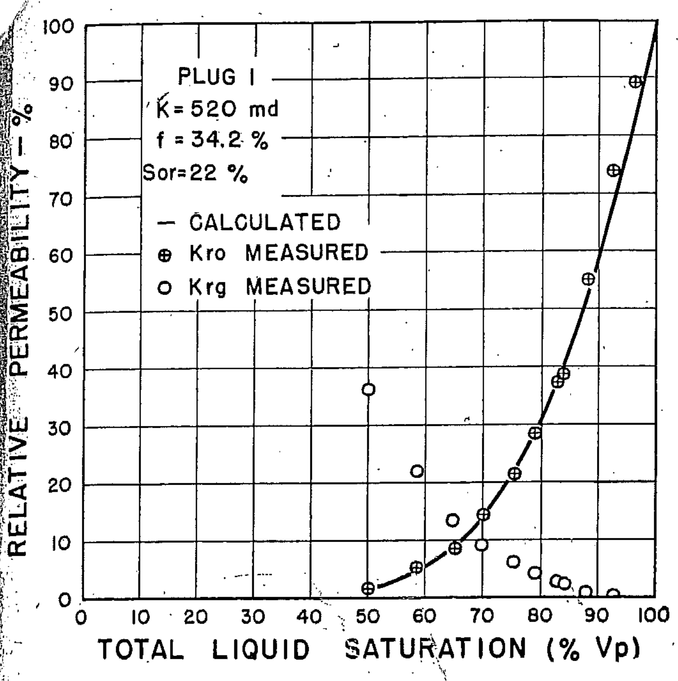

Figure 3—Relative permeabilities on poorly consolidated sand showing agreement with calculated values.

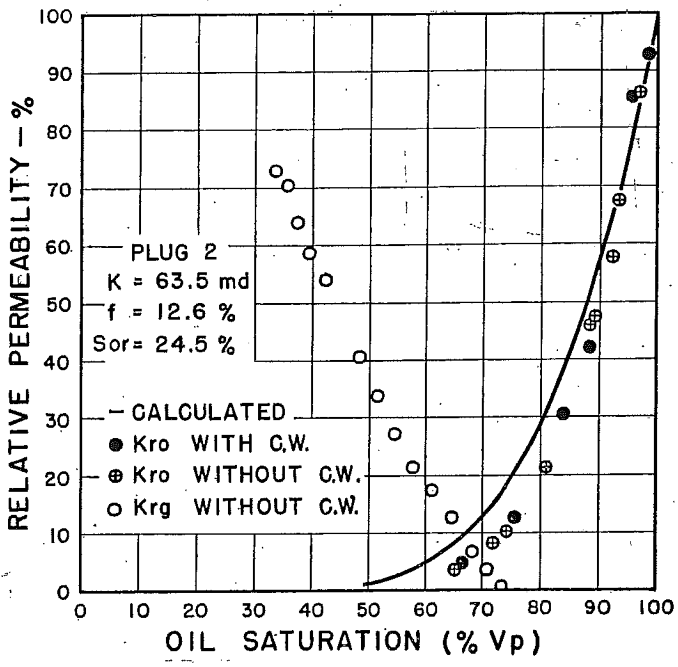

Figure 4—Relative permeabilities on sand cemented with dolomite showing deviations from calculated values.

In equations (1) and (2), S_o is the oil saturation expressed as a fraction of the pore volume; S_{or} is the residual oil saturation; S_m is the lowest oil saturation at which the gas tortuosity is infinite; P_c is the capillary pressure. The expression $\left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^2$ is a parameter analogous to that defined by Gates and Lietz[5] as the tortuosity coefficient, e. The ratios of integrals in equations (1) and (2) are functions of the mean hydraulic radii of the oil and gas channels at any saturation, S_o.

Burdine's tortuosity functions can be evaluated if the saturation S_{or} and S_m are known, and the ratios of integrals can be obtained by integrating the areas under curves of the $1/P_c^2$ vs. saturation function.

Theory

Because the theory and observations on which equations (1) and (2) are based have been thoroughly discussed in the literature,[1, 2, 3, 4, 5, 6, 7] derivations of equations (1) and (2) are not given here. It has been found, however, that as a consequence of the characteristics of the $1/P_c^2$ vs. saturation function, Burdine's equations can be simplified.

The determination of a number of oil-gas capillary pressure curves on sedimentary porous material, and the inspection of others measured elsewhere indicate that to a close approximation:

$$1/P_c^2 = \begin{cases} \bar{C}\,(S_o - S_{or}) & \text{for } S_o > S_{or} \\ 0 & \text{for } S_o < S_{or} \end{cases} \qquad (3)$$

where $\bar{C}$ is a constant. If the saturation is expressed as a fraction of the "effective" pore volume (volume of pores minus the volume of residual oil),

$$1/P_c^2 = C\,S_{oe} \qquad (4)$$

where $C = \bar{C}\,(1 - S_{or})$, and where S_{oe} is the effective saturation defined by

$$S_{oe} = \frac{S_o - S_{or}}{1 - S_{or}} \qquad (5)$$

Because of the linear relationship expressed in equation (4) and illustrated in Fig. 1, the ratios of integrals in equations (1) and (2) can be evaluated from the geometry of similar right triangles. At any saturation, S_{oe}, the ratio applying to the oil phase is S_{oe}^2 and that applying to the gas phase is $1 - S_{oe}^2$. Reference to Fig. 1 will make this situation clear. In Fig. 1, the values A_{70} and A_{100} represent the integrals at the saturations 0.7 and 1.0 respectively.‡ Consequently equation (1) becomes

$$K_{ro} = S_{oe}^4 \qquad (6)$$

Moreover, if S_m is assumed to be unity in a first approximation, equation (2) becomes

$$K_{rg} = (1 - S_{oe})^2\,(1 - S_{oe}^2) \qquad (7)$$

Equations (6) and (7) imply a relationship that should be applicable for the calculation of oil relative permeabilities from measured gas relative permeabilities. Although it is expected that the validity of equations (6) and (7) is confined to natural sediments, the need for capillary pressure data required for the solutions of equations (1) and (2) is avoided. Perhaps the best that can be expected for any generalization of this kind is an approximation. It is desirable that the approximation not require measurements which under some conditions might be as difficult as the direct measurement of relative permeability.

Experimental Procedures

Relative permeability measurements have been made on a large number of cores from several reservoir formations by the capillary pressure technique. The equipment employed is similar to that described by Gates and Lietz,[5] but it has a number of mechanical refinements which make it adaptable to the semi-routine analyses of oil field cores. With this equipment, consistently reproducible results have been obtained on many reservoir materials.

In addition to the capillary pressure technique, a simplified procedure has been developed for the rapid measurement of gas relative permeabilities, which is similar to that designated by Osoba[8] et al. as the "stationary liquid method." The method differs from theirs in that the cores are generally not plastic-coated and are not confined at the time of desaturation. The cores are placed in a sleeve only during actual flow measurements. Oil is removed in increments with an absorbing tissue, and saturations are determined gravimetrically.

Because of the convenience and speed with which gas relative permeabilities can be measured by this technique, the data obtained for gas were used to compute oil relative permeabilities, and these were compared with values measured by the capillary pressure technique.

The procedure was as follows:

(1) Using equation (7), approximate values of S_{oe} are determined corresponding to measured values of K_{rg} and S_o.†

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‡Note that the constant C cancels in the ratio of integrals in equation (1). It is therefore not necessary to relate C to the actual displacement pressure; C can be chosen to give the best possible fit to the capillary pressure curve. Also, because of the smoothing effect of integration, even a capillary pressure curve which deviates considerably from the relation (4) will give values for the ratio of integrals which will be very close to S_{oe}^2.

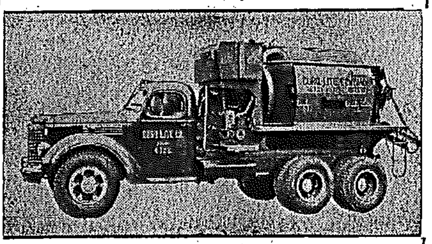
†K_{rg} values measured at very low oil saturations should be ignored because equation (7) is not applicable at saturations less than S_{or}.

(2) These calculated values, which may be designated as $\bar{S}_{oe}$, are plotted as a function of S_o.

If S_m were exactly unity, then $\bar{S}_{oe}$ would equal S_{oe} and, by equation (5), would give a straight line. In fact, $S_m \neq 1$ in general, and the curve is only approximately straight. However, if this curve is extrapolated to $\bar{S}_{oe} = 0$ and $\bar{S}_{oe} = 1$, the corresponding values of S_o are exactly S_{or} and S_m.

(3) Using the value of S_{or} obtained in this way, values of oil relative permeability are obtained from the relationship,

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^4$$



The procedure can be expedited by making an accurate plot of the function

$$K_{rg} = (1 - \bar{S}_{oe})^2 \; (1 - \bar{S}_{oe}^2).$$

By the use of such a plot, complete K_{ro} curves can be obtained from measured K_{rg} data in a few minutes.

Results and Discussion

Altogether, the measured relative permeabilities of about forty cores were analyzed in the manner described above. In about two-thirds of the cases, the measured values of K_{ro} were in good agreement with the theory. Examples are shown in Figures 2 and 3. Only a few cores gave results differing greatly from the measured values. Examples of three different types of variations are shown in Figures 4, 5, and 6. These examples represent the most extreme variations observed. In most cases the variations, if any, were much smaller.

These statistics are probably not particularly significant, however, because many of the cores were from the same formation. From the assumptions made in the derivation of equation (6), it is unlikely that it would apply to materials having unusual pore size distributions such as synthetic materials, aggregated soils, vugular limestones, or sands containing a high percentage of cementing material.

In the derivation, the fluids are presumed not to react with the matrix. Consequently, the method might not apply to water-gas or even water-oil systems. Moreover, there is no evidence to support the use of equation (5) for systems in which a wetting phase replaces a non-wetting phase as in imbibition or a water flood.

Not all of the situations described in the foregoing paragraphs have been examined. Equation (6) failed, however, in the case of sand cores containing considerable dolomitic cementing material. A comparison of measured and calculated data on a sand core of this type is shown in Figure 4. Similar data for a core from the same formation but containing less cementing material are shown in Figure 2.

Equation (6) also failed in the case of cores having pronounced stratification. An extreme example is illustrated in Figure 5. In this case the measured curves contained inflections not predicted by equation (6). Cores from the same formation not having visible stratifications had relative permeabilities in excellent agreement with computed values. This formation consists of poorly consolidated sand containing shale laminations but little cementing material.

The measured oil relative permeability curves sometimes contain an inflection at or near a saturation corresponding to the extrapolated S_m. An analogous phenomenon was observed by Botset[9] on a consolidated sandstone. Oil and gas relative permeability curves on a consolidated sandstone are shown in Figure 6 which illustrate a rather extreme example of this type of inflection.

Wyckoff and Botset[10] defined an "equilibrium saturation" above which they were unable to obtain steady state flow of gas and liquid mixtures in unconsolidated sands. With the techniques employed in this study, however, it is possible to control the pressure gradients in both phases independently, and steady state flow was obtained at all saturations. Whether or not the saturations obtained were uniform on a microscopic or even macroscopic basis or whether the systems were at all times in a state of capillary equilibrium are questions which are too complex to be discussed in this paper.

It should be noted that the term S_m as previously defined is not necessarily equal to the "equilibrium saturation." In fact, S_m may have no physical significance except that implicit in the method of its derivation.

A factor which did not have any measurable effect on oil relative permeabilities was the presence or absence of residual brine. This situation is illustrated in Figures 2 and 4.

Equation (7) has not been used except as a tool for getting the saturation, S_{or}, needed for the solution of equation (6). It might be useful for obtaining K_{rg} curves also if a method could be found for determining S_{or} and S_m that is more convenient than measuring gas relative permeabilities.

Summary and Conclusions

Measurements on a large number of cores indicate that a definite relationship usually exists between gas and oil relative permeabilities. The observed relationship can be deduced from the characteristics of capillary pressure-desaturation curves obtained on porous sedimentary rock.

Examination of capillary pressure-desaturation curves leads to the conclusion that to a close approximation:

$$1/P_c^2 = C\left(\frac{S_o - S_{or}}{1 - S_{or}}\right)$$

From this relationship based entirely on observation and from the findings of Burdine concerning the nature of the tortuosity-saturation function, an expression for oil relative permeability was derived.

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^4$$

and for gas relative permeability:

$$K_{rg} = \left[1 - \left(\frac{S_o - S_{or}}{S_m - S_{or}}\right)\right]^2 \left[1 - \left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^2\right]$$

Using gas relative permeabilities measured by a simple and rapid technique, values of a residual oil saturation, S_{or}, were obtained by means of which K_{ro} curves were calculated.

The calculated K_{ro} curves were in good agreement with measured values on many cores from a number of reservoir formations. From an analysis of the assumptions made in the derivation of the relationships presented, it would seem that their validity would be confined to reservoir materials not having marked stratifications, large quantities of cementing material, or large solution channels.

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REFERENCES

[1] Rose, W. D., and Bruce, W. A., Trans. AIME, 186, 127 (1949).

[2] Thornton, O. F., Trans. AIME, 186, 328 (1949).

[3] Wyllie, M. R. J., and Rose, W. D., Trans. AIME, 189, 105 (1950).

[4] Purcell, W. R., TRANS. AIME, 186, 39 (1949).

[5] Gates, J. I., and Lietz, W. T., API Drilling and Production Practice (1950).

[6] Wyllie, M. R. J., and Spangler, M. B., Bull. AAPG, 36, 359 (1952).

[7] Burdine, N. T., J. PET. TECH., No. 3 (March, 1953).

[8] Osoba, J. S.; Richardson, J. G.; Kerrer, J. K.; Hafford, J. A.; and Blair, P. M., Trans. AIME, 192, 47 (1951).

[9] Botset, H. G., Trans. AIME, 127, 226 (1938).

[10] Wyckoff, R. D., and Botset, H. G., PHYSICS, 7, 325 (1936).

Diamond Alkali Co. Buys Muscle Shoals Chl. Plant

The Muscle Shoals chlorine-caustic soda plant, recently sold by the government to Diamond Alkali Company, is now being put into condition for operation and will be in production by Jan. 1, 1955.

John A. Sargent, president of this major producer of basic chemicals, made the above statement to a gathering of more than 100 civic and industrial leaders of Sheffield, Florence, Tuscumbia, and Muscle Shoals City at a meeting in October at the Tennessee Valley Golf and Country Club.

Diamond officially took possession of the plant at a brief ribbon-cutting ceremony when A. H. Ingley, vice president in charge of manufacturing for the company, presented a check for $1,950,000—part payment on the purchase price of $15,127,000—to Brig. Gen. Charles G. Holle, Division Engineer, Corps of Engineers, U. S. Army, Atlanta.

Discussing Diamond's plans for the multi-million-dollar plant, one of the largest, most modern and complete installations of its kind, Sargent said:

"The diversity of our four other chlorine-caustic soda producing points—at Pine Bluff, Arkansas; Houston, Texas; Painesville, Ohio; Edgewood, Maryland—gives Diamond the advantage of flexibility—a factor holding strong importance in these days of increasingly keen competition.

Sargent noted one very important development in this connection.

"As a result of the Muscle Shoals chlorine-caustic soda plant becoming established as a principal producing plant for these two vital chemicals," he declared, "it is reasonable to expect that customers of these products, in the area served by this plant, will enjoy lower delivered costs on these products.

"Basic to many products and processes, caustic soda is especially important to the petroleum industry," Sargent said. "Chlorine is essential for water purification, and for the production of bleaching materials for paper and textiles as well as the manufacture of drugs, other chemicals, dyes, refrigerants, insecticides, and weed-killers among other products" he added.

Modern Fluids Test Laboratory At University of Cincinnati

University of Cincinnati Department of Civil Engineering headed by Professor Cornelius Wandmacher, recently completed construction of a modern Fluids Laboratory.

Designed and constructed almost entirely by campus personnel it is an outstanding example of advancement in contemporary engineering education.

Skillful planning has resulted in a modern instructional facility where classroom theories are effectively transformed into the realm of practical application through lecture, demonstration and experimentation.

An outstanding feature is the extensive use of glass and plastic to allow the students to observe the behavior of different fluids under various conditions. Color coding of all the overhead pipes and fittings aids in tracing the flow.

Principal equipment in the laboratory includes an oil flow unit to demonstrate both laminar and turbulent flow, a 24-foot glass-sided flume, a combination Reynolds number and orifice tank, a modern water turbine, a water hammer and surge unit for studying transient pressures, models of hydraulic structures and various meters, gages and piping set-ups.

Worthington Corporation, one of the world's largest producers of pumps, has supplied the pumping equipment for the laboratory. This includes a vertical turbine pump which operates at 1,750 RPM with a 500 GPM rated capacity at a 65 foot head.

A general purpose end-suction horizontal pump of the volute type was also supplied by Worthington. This pump operates at 3,600 RPM with a 200 GPM capacity at 150 foot head.

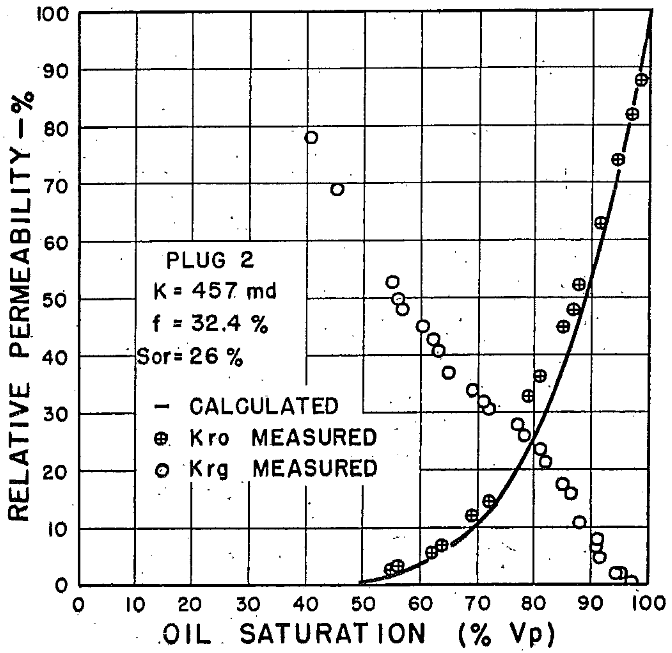

Figure 5—Relative permeabilities on poorly consolidated sand with stratifications showing deviations from calculated values.

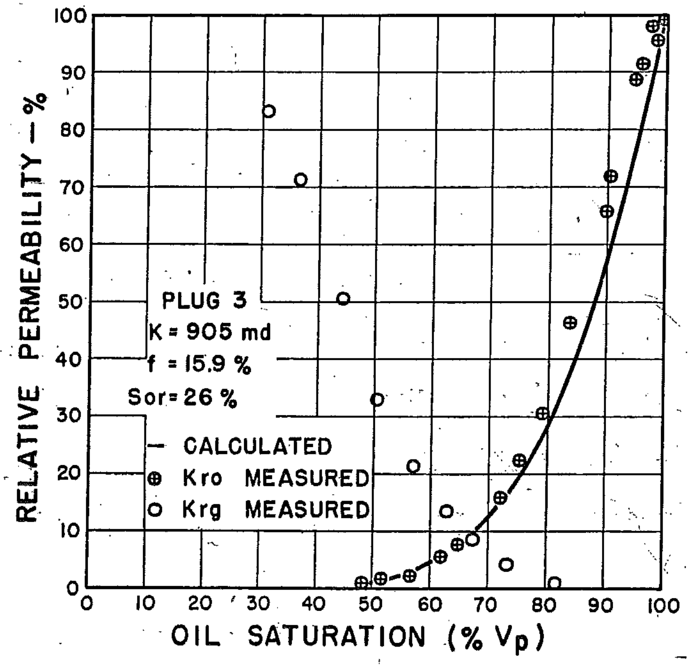

Figure 6—Relative permeabilitiese on consolidated sand showing deviations from calculated values in region of low gas saturation.